Midterm Exam Calculus 1

27 september 2006, 9.00-11.00.

The use of annotations, books and calculators is not permitted in this examination.

1. Explain why $\log_3(1/2)$ can not be written as t/n with t and n integer.

2. Use the definition of 'limit' to show that

$$\lim_{h \to 2} x^2 - 2x = 0$$

3. The function $f : \mathbb{R} \to \mathbb{R}$ is continuous at $x = 0$. Prove that the function g, that is given by $g(x) = xf(x)$, is differentiable at $x = 0$

4. The function f is given by $f(x) = (x^2 + 2x - 8)^3$. Find all real numbers x such that $f'(x) = 0$.

5. Given a real number n and the function $f(x) = x^n$ (for $x > 0$). The line through the middle of the segment with endpoints $(0,0)$ and $(t, f(t))$, which also is perpendicular to the line segment, intersects the y-axis at a height $y(t)$. If it is given that $\lim_{t \downarrow 0} y(t)$ exists (as a real number), what can you say about n?